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**DESIGN CURVES FOR THE BUCKLING
OF SANDWICH CYLINDERS OF FINITE
LENGTH UNDER UNIFORM EXTERNAL
LATERAL PRESSURE**

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DESIGN CURVES FOR THE BUCKLING OF SANDWICH CYLINDERS OF
FINITE LENGTH UNDER UNIFORM EXTERNAL LATERAL PRESSURE^{1,2}

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Abstract

This report contains curves and formulas for the calculation of the critical external pressure of finite length, circular cylindrical shells with sandwich walls. The facings of the sandwich are equal and isotropic and their individual stiffness is not taken into account. The sandwich core is isotropic or orthotropic having natural axes in the axial, tangential, and radial directions of the cylinder. Curves are given for cylinders with sandwich walls having isotropic cores and orthotropic cores having certain relative elastic properties. If the cores are very rigid, the method yields results that are substantially those of von Mises.

¹-This Note is a revision of a report by the same title, issued May 1959 as Forest Products Laboratory Report No. 1869. This revision was made by E. W. Kuenzi and B. Bohannon.

²-This progress report is one of a series (ANC-23, Item 57-3) prepared and distributed by the Forest Products Laboratory under U. S. Navy; Bureau of Naval Weapons Order No. 19-61-8041 WEPS and U. S. Air Force Contract No. DO 33(616)61-06. Results here reported are preliminary and may be revised as additional data become available.

³-Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

Introduction

This report presents design curves for the critical external radial pressure of circular cylindrical shells with sandwich walls, calculated according to the formulas developed in Forest Products Laboratory Report 1844-B (9)⁴. The sandwich cylinder walls have isotropic facings and orthotropic or isotropic cores. The natural axes of the orthotropic cores are axial, tangential, and radial. These formulas reduce substantially to those developed by von Mises (7, 10, 14) when the core is very rigid.

A great deal of investigative work has been done on isotropic cylindrical shells subjected to external pressure since report 1844-B giving theoretical analysis of sandwich cylinders was written. It has been found that experiment sometimes yields critical loads that are less than those predicted by von Mises' theory (13). This has been attributed to two causes. First, the experimental cylinders contained imperfections that lowered the critical load (1, 2, 3, 8, 12). Second, lower energy levels are associated with post-buckling configurations of the cylinder than with those just at buckling, the former being reached without the necessity of passing through (snap-through) the latter or the energy necessary for passing through the latter being supplied by vibration or shocks (4, 5, 6, 8).

The curves published in this report do not consider snap-through buckling or cylinders with imperfections. Sandwich cylinders, however, are much more perfect than their solid counterparts because they are thicker and the effect of an imperfection is in proportion to the ratio of its amplitude to the thickness of the cylindrical shell. Also, the curves neglect the stiffnesses of the individual facings. These stiffnesses add to the critical loads when the cylinders are short, and it is for short cylinders that "snap-through" is likely to occur (6).

Development of Formula from Which Design Curves Were Calculated

The critical external pressure is found by solving equation (51) on page 23 of Report No. 1844-B (9) for the buckling coefficient. The determinant of equation (51) can be simplified if the transverse modulus of elasticity of the core (E_c) is assumed to be infinite. For most core materials except possibly for low density foams E_c is sufficiently large so that this assumption yields values of the critical pressure that are only very slightly too great.

⁴Underlined numbers in parentheses refer to the references.

Before $\underline{E_c}$ is allowed to approach infinity, the first and fourth columns of the determinant are multiplied by $\frac{G_{Rz}}{\underline{E_c}}$ and the third column is divided by $\underline{E_c}$.

Then when $\underline{E_c}$ approaches infinity, the expressions in rows 3, 4, 5, and 6 in column 3 approach zero.

The expressions in each row, excepting the first, are replaced by new expressions, as indicated by the following formulas in which \underline{R} represents the expression in the row designated by its subscript and in some column. The primed values are the new ones to be substituted for the old.

$$R_2' = R_2 \frac{a^2}{b^2} + R_1$$

$$R_3' = R_3 + R_2'$$

$$R_4' = R_4 + R_2'$$

$$R_5' = 2R_5 + (n^2 + 3 \lambda^2) R_2'$$

$$R_6' = 2R_6 + (n^2 + 3 \frac{b^2}{a^2} \lambda^2) R_2'$$

These substitutions cause the expressions in column 3 and rows 2, 3, 4, 5, and 6 and those in column 6 and rows 3, 4, 5, and 6 to become zero, and the 6-by-6 determinant is readily reduced (by minors) to a 4-by-4 determinant. This determinant is simplified slightly by replacing the second row of expressions by the second row minus the first row.

After the determinant was written in the above form, a change in parameters was made using the following nomenclature:

d -- thickness of the sandwich

E -- modulus of elasticity of facing

G_{Rz} -- modulus of rigidity of core in the radial and axial directions

$G_{R\theta}$ -- modulus of rigidity of core in the radial and tangential directions

h -- distance between facing centroids

$$k -- \frac{qr(1 - \mu^2)}{2Et}$$

L -- length of cylinder

n -- number of half waves in the circumference of the cylinder

q -- the external critical pressure on the cylinder

r -- mean radius of the sandwich cylinder

$$R -- \frac{G_{Rz}}{G_{R\theta}}$$

t -- thickness of the facings

$$V -- \frac{Et}{4r(1 - \mu^2) G_{R\theta}}$$

μ -- Poisson's ratio

The radii \underline{a} and \underline{b} were eliminated by the following equations obtained from the geometry of the cylinder.

$$a = r + \frac{h}{2}$$

$$b = r - \frac{h}{2}$$

and the following substitutions were made:

$$\Phi = \frac{4r}{d}$$

$$\phi = \frac{2h}{d}$$

$$\mu = \frac{1}{3}$$

The resulting expressions in the 4-by-4 determinant are:

Row 1, column 1

$$\frac{n^2 - 1}{n^2} \frac{\Phi + \phi}{\Phi - \phi} - 1 + 2V \left(\frac{\Phi}{\Phi + \phi} \right) \left[1 - \frac{1}{3n^2} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right]$$

Row 2, column 1

$$\frac{2}{n^2} \frac{\Phi}{\Phi - \phi} + \frac{8V}{(\Phi - \phi) \left(\frac{\Phi^2}{\phi^2} - \phi^2 \right)}$$

Row 3, column 1

$$\begin{aligned} & \frac{n^2 - 1}{n^2} \frac{2\phi}{\Phi - \phi} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] + 4V \frac{\Phi + \phi}{\Phi} \left(\frac{\pi r}{L} \right)^2 \left[\frac{1}{3} \right. \\ & \left. - \frac{1}{n^2} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 - k \frac{\Phi^2 - \phi^2}{\Phi^2} \right] \end{aligned}$$

Row 4, column 1

$$\begin{aligned} & \frac{n^2 - 1}{n^2} \frac{2\phi}{\Phi - \phi} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \right] + 4V \frac{\Phi + \phi}{\Phi} \left(\frac{\pi r}{L} \right)^2 \left[\frac{1}{3} \right. \\ & \left. - \frac{1}{n^2} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 - k \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] \end{aligned}$$

Row 1, column 2

$$2k (n^2 - 1) \left(\frac{\Phi + \phi}{\Phi} \right)^2 \frac{\Phi^2 + \phi^2}{\Phi^2 - \phi^2} - n^2 + 1 + \frac{1}{3} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2$$

Row 2, column 2

$$- \frac{2\phi}{\Phi + \phi} \left[(n^2 - 1) \frac{\Phi + \phi}{\Phi - \phi} + \frac{1}{3} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right]$$

Row 3, column 2

$$\begin{aligned} & 2 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \left[- \frac{n^2 - 1}{3} + \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 + k (n^2 - 1) \frac{\Phi^2 - \phi^2}{\Phi^2} \right] \\ & + 2k (n^2 - 1) \left(\frac{\Phi + \phi}{\Phi} \right)^2 \frac{\Phi^2 + \phi^2}{\Phi^2 - \phi^2} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] \end{aligned}$$

Row 4, column 2

$$2 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \left[-\frac{n^2 - 1}{3} \frac{\Phi - \phi}{\Phi + \phi} + \left(\frac{\pi r}{L} \right)^2 \frac{(\Phi - \phi)^3}{\Phi^2 (\Phi + \phi)} \right. \\ \left. + k (n^2 - 1) \frac{\Phi^2 - \phi^2}{\Phi^2} \right] + 2 k (n^2 - 1) \left(\frac{\Phi + \phi}{\Phi} \right)^2 \frac{\Phi^2 + \phi^2}{\Phi^2 - \phi^2} \left[n^2 \right. \\ \left. + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \right]$$

Row 1, column 3

$$\frac{2\phi}{\Phi + \phi} + \frac{4V}{3R} \frac{\Phi}{\Phi + \phi} \log \frac{\Phi - \phi}{\Phi + \phi}$$

Row 2, column 3

$$- \frac{8V}{3R} \frac{\Phi}{\Phi + \phi} \log \frac{\Phi - \phi}{\Phi + \phi}$$

Row 3, column 3

$$- 2 + \frac{2\phi}{\Phi + \phi} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] + \frac{4V}{3R} \frac{\Phi}{\Phi + \phi} \left[n^2 \right. \\ \left. + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] \log \frac{\Phi - \phi}{\Phi + \phi}$$

Row 4, column 3

$$2 \frac{\Phi - \phi}{\Phi + \phi} + \frac{2\phi}{\Phi + \phi} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \right] - \frac{4V}{3R} \frac{\Phi}{\Phi + \phi} \left[n^2 \right. \\ \left. + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \right] \log \frac{\Phi - \phi}{\Phi + \phi}$$

Row 1, column 4

$$n^2 - \frac{1}{3} \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2$$

Row 2, column 4

$$- \frac{4}{3} \left(\frac{\pi r}{L} \right)^2 \frac{\phi}{\Phi}$$

Row 3, column 4

$$4 n^2 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \left[\frac{1}{3} - \frac{k}{2} \frac{\Phi^2 - \phi^2}{\Phi^2} \right] - \frac{2}{3} \left(\frac{\pi r}{L} \right)^2 \frac{\Phi^2 + \phi^2}{\Phi^2} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right]$$

Row 4, column 4

$$4 n^2 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \left[\frac{1}{3} - \frac{k}{2} \left(\frac{\Phi + \phi}{\Phi} \right)^2 \right] - \frac{2}{3} \left(\frac{\pi r}{L} \right)^2 \frac{\Phi^2 + \phi^2}{\Phi^2} \left[n^2 + 3 \left(\frac{\pi r}{L} \right)^2 \left(\frac{\Phi - \phi}{\Phi} \right)^2 \right]$$

By setting this determinant equal to zero and solving for \underline{k} , the critical pressure can be found by:

$$q = \frac{2 E t}{r (1 - \mu^2)} k$$

This represents a theoretical solution for the critical pressure with the assumption that the sandwich core modulus of elasticity is infinite.

The upper and lower limits for k can be determined as given in Forest Products Laboratory Reports 1844-A and 1844-B. These limits are for cylinders with membrane facings and a core with a modulus of elasticity that is infinite. The lower limit is approached as the length of cylinders becomes infinite and is given by equation (72) of Report 1844-A as

$$k = \frac{3 (\Phi - \phi) \phi}{16 (\Phi^2 + \phi^2) \left[\frac{\Phi^2 - \phi^2}{16 \Phi \phi} + V \right]}$$

The upper limit for \underline{k} is reached as \underline{n} becomes infinite; it is

$$k = \frac{\phi}{4 (\underline{\Phi} + \phi) V}$$

which corresponds to the critical pressure as given on page 28 of Report 1844-B.

From this critical pressure the critical hoop compression per unit length of cylinder is:

$$N_{\theta} = k G_{R\theta}$$

which is the usual limit imposed on the edge compression of sandwich constructions with membrane facings by the shear instability of the core.

The above determinant can be further simplified by a few approximations without any significant loss of accuracy. Examination of the parameters $\underline{\Phi}$ and ϕ shows that $\underline{\Phi}$ is large in comparison to ϕ . Therefore, any terms of $(\underline{\Phi} \pm \phi)$ can be assumed as equal to $\underline{\Phi}$ with only a minor loss of accuracy. Making this assumption and substituting $\beta = \left(\frac{\pi r}{L}\right)^2$, the determinant reduces to the following expressions:

Row 1, column 1

$$- \frac{1}{n^2} + 2 V \left[1 - \frac{\beta}{3n^2} \right]$$

Row 2, column 1

$$\frac{2}{n^2} + 8 V \frac{\phi}{\underline{\Phi}}$$

Row 3, column 1

$$\frac{n^2 - 1}{n^2} \frac{2 \phi}{\underline{\Phi}} \left[n^2 + 3 \beta \right] + 4 V \beta \left[\frac{1}{3} - \frac{\beta}{n^2} - k \right]$$

Row 4, column 1

$$\frac{n^2 - 1}{n^2} \frac{2\phi}{\Phi} \left[n^2 + 3\beta \right] + 4V\beta \left[\frac{1}{3} - \frac{\beta}{n^2} - k \right]$$

Row 1, column 2

$$\left(n^2 - 1 \right) \left(2k - 1 \right) + \frac{\beta}{3}$$

Row 2, column 2

$$- \frac{2\phi}{\Phi} \left[\left(n^2 - 1 \right) + \frac{\beta}{3} \right]$$

Row 3, column 2

$$2\beta \left[- \frac{n^2 - 1}{3} + \beta + k \left(n^2 - 1 \right) \right] + 2k \left(n^2 - 1 \right) \left[n^2 + 3\beta \right]$$

Row 4, column 2

$$2\beta \left[- \frac{n^2 - 1}{3} + \beta + k \left(n^2 - 1 \right) \right] + 2k \left(n^2 - 1 \right) \left[n^2 + 3\beta \right]$$

Row 1, column 3

$$\frac{2\phi}{\Phi}$$

Row 2, column 3

$$0$$

Row 3, column 3

$$- 2 + \frac{2\phi}{\Phi} \left[n^2 + 3\beta \right]$$

Row 4, column 3

$$2 + \frac{2\phi}{\Phi} \left[n^2 + 3\beta \right]$$

Row 1, column 4

$$n^2 - \frac{\beta}{3}$$

Row 2, column 4

$$- \frac{4\beta}{3} \frac{\phi}{\Phi}$$

Row 3, column 4

$$4n^2\beta \left[\frac{1}{3} - \frac{k}{2} \right] - \frac{2\beta}{3} \left[n^2 + 3\beta \right]$$

Row 4, column 4

$$4n^2\beta \left[\frac{1}{3} - \frac{k}{2} \right] - \frac{2\beta}{3} \left[n^2 + 3\beta \right]$$

By setting this determinant equal to zero, the approximate solution for \underline{k} is:

$$\underline{k} = \frac{\frac{8}{9} + \alpha^2 \left(n^2 - 1 \right) \left(3 + \frac{n^2}{\beta} \right) \left[\left(\frac{n^2}{\beta} - \frac{1}{3} \right) \left(n^2 - 1 + \beta \right) - \frac{2}{3} \right] + \frac{32}{9} \sqrt{\alpha \left(n^2 + \frac{\beta}{3} \right)}}{\left[\left(n^2 - 1 \right) \left(1 + \frac{n^2}{\beta} \right)^2 + \frac{1}{3} \right] \left[1 + 4 \sqrt{\alpha \left(n^2 + \frac{\beta}{3} \right)} \right]}$$

$$\text{where } \alpha = \frac{\phi}{\Phi} = \frac{h}{2r}$$

To obtain this value of \underline{k} , terms containing \underline{k}^2 and $\underline{k} \alpha^2$ were neglected. It was also assumed that terms $(1 \pm \underline{m} \alpha) = 1$ where \underline{m} is a small whole number.

Further simplifications can be made by neglecting the terms of $+\frac{1}{3}$, $-\frac{1}{3}$, $-\frac{2}{3}$, and $\frac{\beta}{3}$ so that the final approximate solution for \underline{k} is:

$$k = \frac{\frac{8}{9} + \alpha^2 (n^2 - 1) \left(3 + \frac{n^2}{\beta}\right) \left(\frac{n^2}{\beta}\right) (n^2 - 1 + \beta) + \frac{32}{9} V \alpha n^2}{(n^2 - 1) \left(1 + \frac{n^2}{\beta}\right)^2 \left[1 + 4 V \alpha n^2\right]}$$

The limits of this approximate solution are the same as for the exact solution if the terms $(\Phi \pm \phi)$ are assumed equal to $\underline{\Phi}$.

Description of Design Curves

Using the approximate equation, values for k were determined for various values of \underline{V} , \underline{n} , $\frac{\underline{L}}{\underline{r}}$, and $\underline{\alpha}^2$. These values of k are plotted (figs. 1 - 5) for values of $\frac{\underline{L}}{\underline{r}}$ ranging from 1 to 100. Curves are given for values of \underline{V} equal to 0, 0.5, 1.0, 2.0, and 4.0. For each of these values of \underline{V} , curves are given for seven values of $\underline{\alpha}^2$ ranging from 10^{-6} to 10^{-4} . These curves apply to sandwich cylinders with equal isotropic facings and isotropic cores or orthotropic cores with their natural axes parallel to the axial, tangential, and radial directions of the cylinders. The critical pressure is given by

$$q = \frac{2 E t}{(1 - \mu^2) r} k$$

From a solution of the exact determinant, it was found that the modulus of rigidity of the core associated with the radial and axial directions $\underline{G_{Rz}}$ has very little influence on the critical pressure. It does not enter the formulas for the limits of critical pressure. The ratio $R = \frac{\underline{G_{Rz}}}{\underline{G_{R\theta}}}$ does affect the critical pressure for small $\frac{\underline{L}}{\underline{r}}$ values but for reasonable values of \underline{R} the critical pressure is affected only slightly. The ratio, \underline{R} , does not appear in the approximate solution for k .

The accuracy of the approximate solution was checked by making a comparison of the approximate and exact solutions for values of \underline{V} equal to 0, 0.5, and 1.0, $\frac{\underline{r}}{\underline{d}}$ equal to 50, and $\frac{\underline{t}}{\underline{d}}$ equal to 1. This comparison is shown on the top curve of

figures 1, 2, and 3 where the small circles represent the exact solution. This shows that, except for small $\frac{L}{r}$ values, the error encountered with the approximate solution is less than 1 percent. For larger values of \underline{V} and $\frac{r}{d}$ the error is less than is shown here. For the small values of $\frac{L}{r}$, it is doubtful that either solution gives the correct results.

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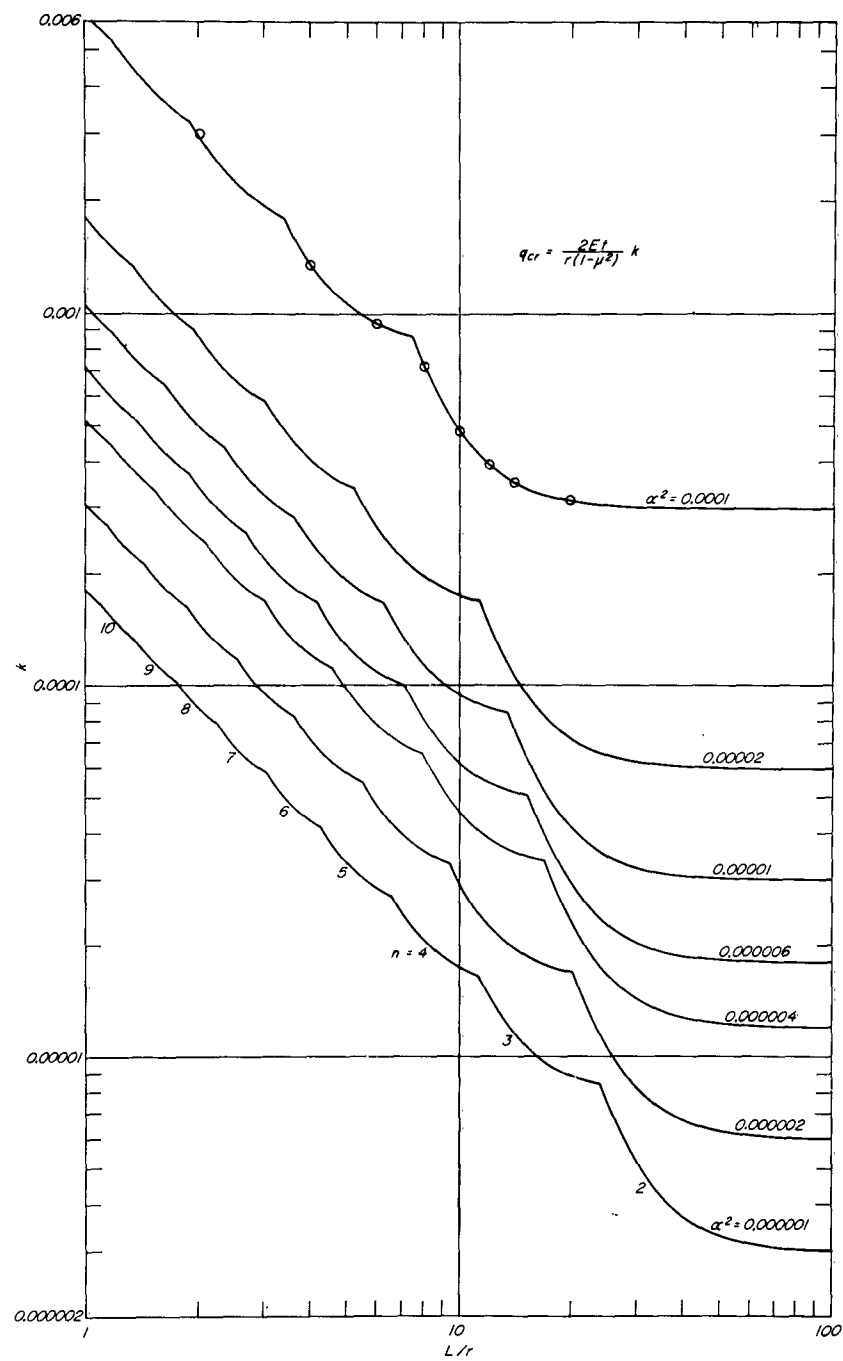


Figure 1.--Values of k for $V = 0$.

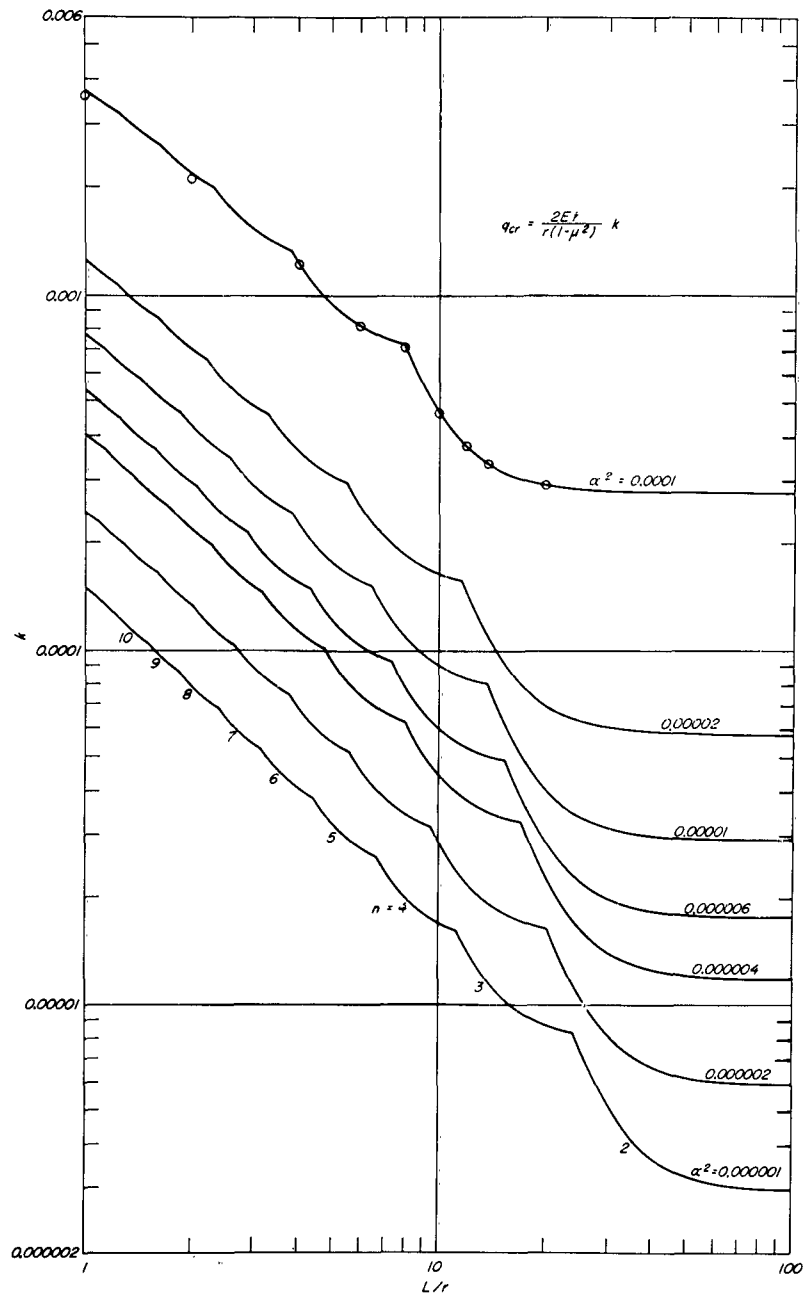


Figure 2.--Values of k for V = 0.5.

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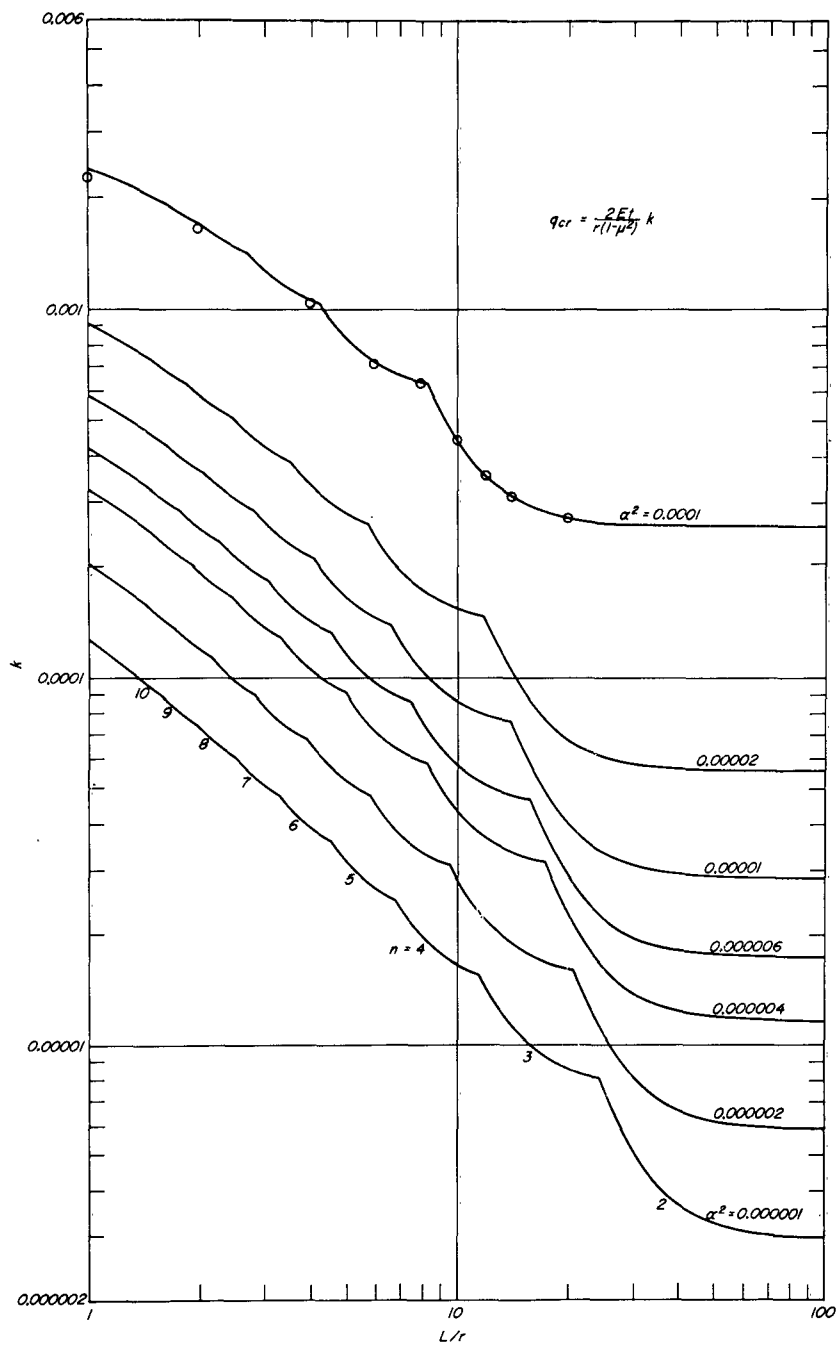


Figure 3.--Values of k for $V = 1.0$.

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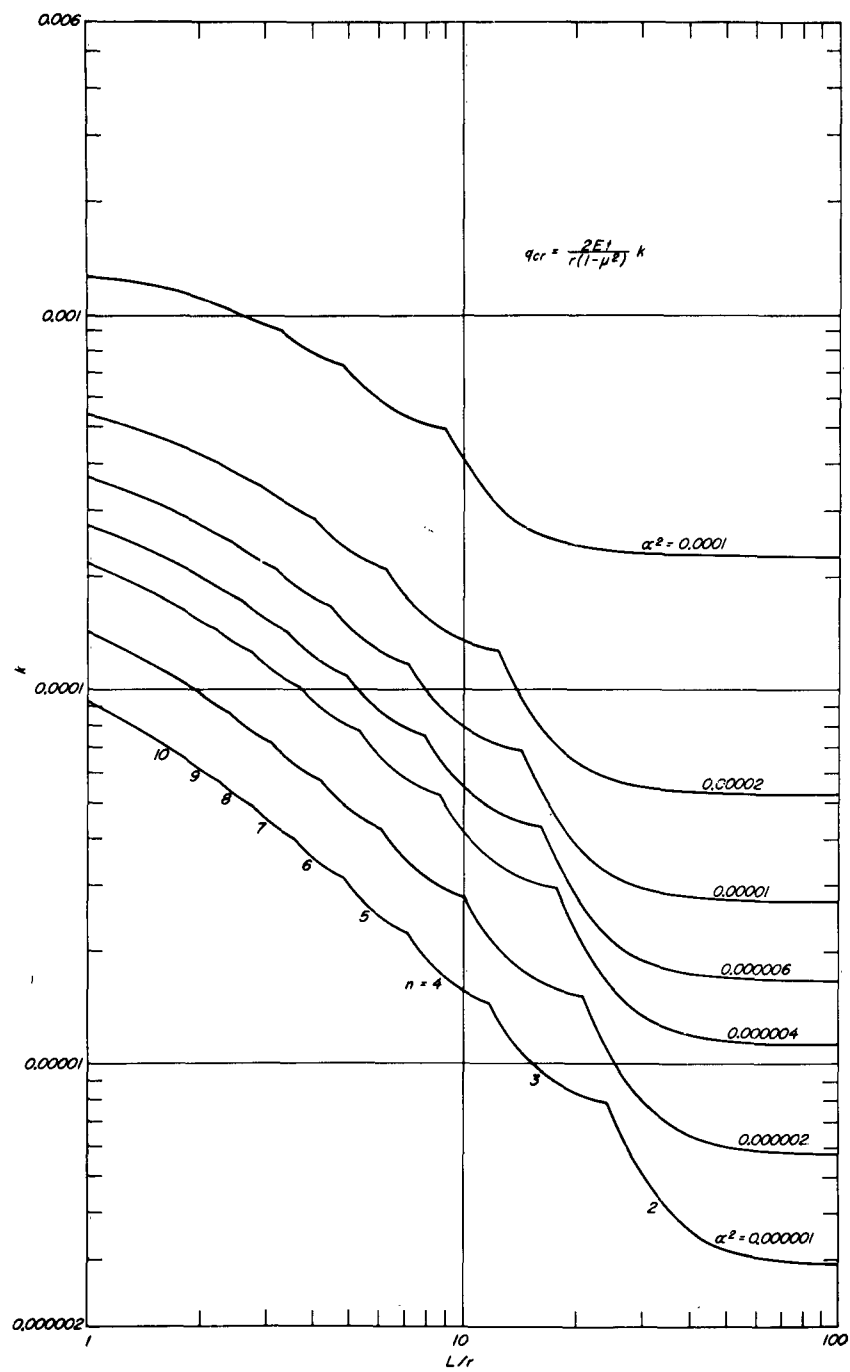


Figure 4. --Values of k for $V = 2.0$.

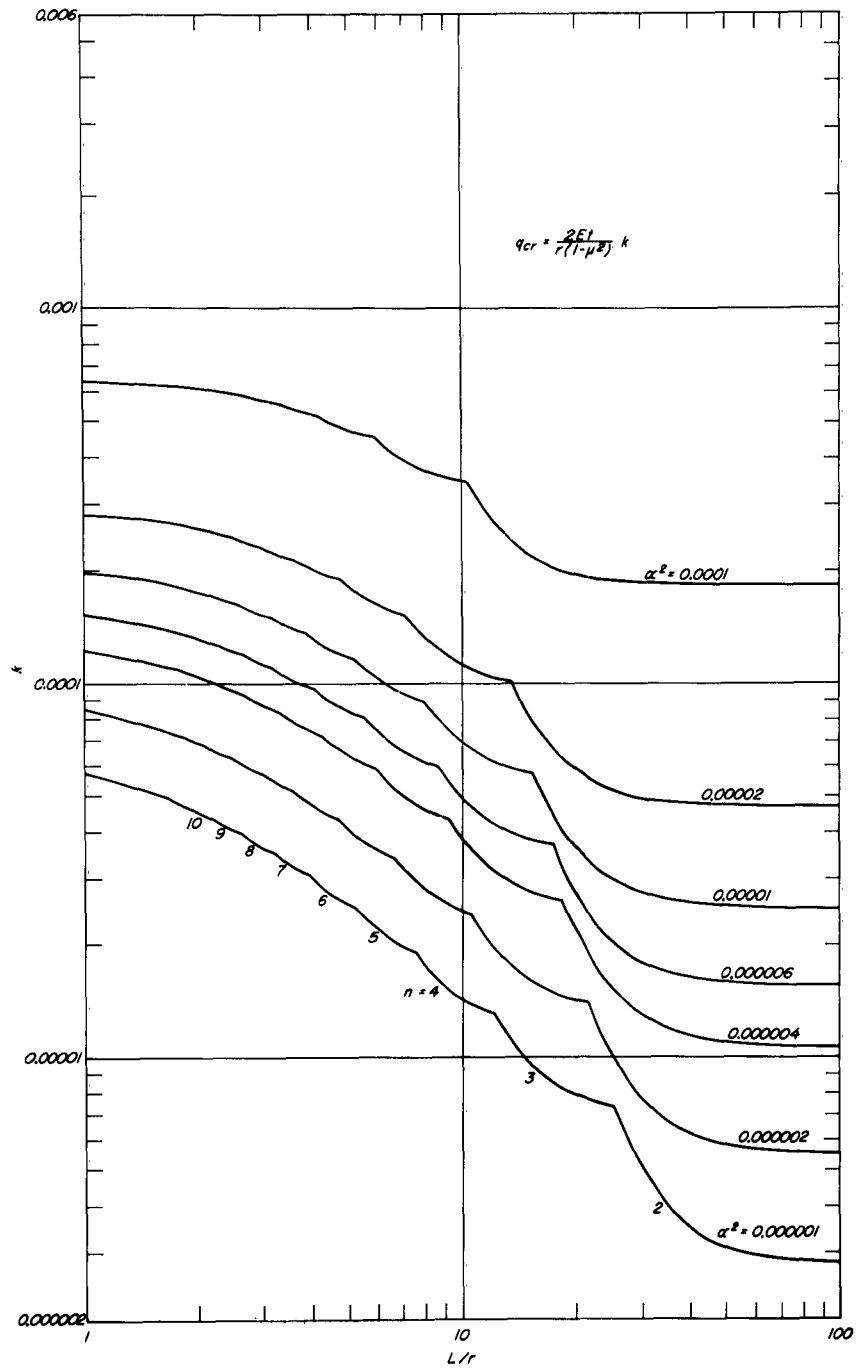


Figure 5.--Values of k for V = 4.0.

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Note: Since Forest Products Laboratory publications are so varied in subject, no single list is issued. Instead a list is made up for each Laboratory division. Twice a year, December 31 and June 30, a list is made up showing new reports for the previous 6 months. This is the only item sent regularly to the Laboratory's mailing list. Anyone who has asked for and received the proper subject lists and who has had his name placed on the mailing list can keep up to date on Forest Products Laboratory publications. Each subject list carries descriptions of all other subject lists.

FPL FILING SYSTEM DESIGNATION--S-5-5

<p>Norris, Charles Brazer Design curves for the buckling of sandwich cylinders of finite length under uniform external lateral pressure, by C. B. Norris and John J. Zahn. Revised by E. W. Kuenzi and B. Bohannan. Madison, Wis., U. S. Forest Products Laboratory, 1963. 14 p., illus. (U.S. FS res. note FPL-07)</p> <p>Contains curves and formulas for the calculation of the critical external pressure of finite length, circular cylindrical shells with sandwich walls. The facings of the sandwich are equal and isotropic and their individual stiffness is not taken into account. The sandwich core is isotropic or orthotropic having natural axes in the axial, tangential, and radial directions of the cylinder. Curves are given for cylinders with sandwich walls having isotropic cores and orthotropic cores having certain relative elastic properties.</p>	<p>Norris, Charles Brazer Design curves for the buckling of sandwich cylinders of finite length under uniform external lateral pressure, by C. B. Norris and John J. Zahn. Revised by E. W. Kuenzi and B. Bohannan. Madison, Wis., U. S. Forest Products Laboratory, 1963. 14 p., illus. (U.S. FS res. note FPL-07)</p> <p>Contains curves and formulas for the calculation of the critical external pressure of finite length, circular cylindrical shells with sandwich walls. The facings of the sandwich are equal and isotropic and their individual stiffness is not taken into account. The sandwich core is isotropic or orthotropic having natural axes in the axial, tangential, and radial directions of the cylinder. Curves are given for cylinders with sandwich walls having isotropic cores and orthotropic cores having certain relative elastic properties.</p>
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